Final exam replaces lowest test grade

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1 Formula

Consider the scenario where a student has a class where the final exam score replaces the lowest test score. There is a separate test category and final category. This analysis assumes that the weight of the test category is divided equally among exams.

$$W_T = \text{Weight of the test category}$$
$$W_F = \text{Weight of the final exam}$$
$$N = \text{The number of tests taken}$$
$$T_n = \text{The score on the nth test}$$
$$F = \text{Score on the final exam}$$
$$C = \text{Current overall grade}$$
$$G = \text{Goal overall grade}$$

Since most students know their current overall grade, we've chosen to include it as a variable. This calculation can also be done with knowledge of the individual categories, in a similar way. By definition:

$$G = (1 - W_F)C - W_T(\frac{\sum T_n}{N}) + W_T(\frac{(\sum T_n) - \min(T_n) + F}{N}) + W_F F$$
$$= (1 - W_F)C + W_T(\frac{F - \min(T_n)}{N}) + W_F F$$

As you can see, the resulting formula is fairly simple. The final exam can be thought of as a separate category, as well as a small point boost of $F - \min(T_n)$ to the lowest test grade, $\min(T_n)$.

And now, solving for F:

$$G = (1 - W_F)C + W_T(\frac{F - \min(T_n)}{N}) + W_F F$$
$$= (1 - W_F)C + \frac{W_T F}{N} - \frac{W_T \min(T_n)}{N} + W_F F$$
$$= (1 - W_F)C - \frac{W_T \min(T_n)}{N} + (\frac{W_T}{N} + W_F)F$$
$$F = \frac{G - (1 - W_F)C + \frac{W_T \min(T_n)}{N}}{\frac{W_T}{N} + W_F}$$
$$F = \frac{GN - (1 - W_F)NC + W_T \min(T_n)}{W_T + NW_F}$$

2 Example

Let's consider the following example:

$$W_T = 75\%$$
$$W_F = 20\%$$
$$N = 6$$
$$T_n = \{80\%, 95\%, 100\%, 100\%, 78\%, 91\%\}$$
$$F = ?$$
$$C = 91.25\%$$
$$G = 90\%$$

Substituting the numbers into the equation:

$$F = \frac{GN - (1 - W_F)NC + W_T \min(T_n)}{W_T + NW_F}$$
$$F = \frac{(0.90)(6) - (1 - 0.20)(6)(0.9125) + (0.75)(0.78)}{0.75 + (6)(0.20)}$$
$$F = 0.823$$

In this example, F, the score needed on the final exam, is 82.3%.