Final Grade Calculator Formulae

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1 Introduction

This document explains the formulae used on the RogerHub Final Grade Calculator. Please direct errata to postmaster at rogerhub.com.

2 Grade required on the final exam

The original Final Grade Calculator, created in 2009, only performed one operation. Given the following variables, it would determine the minimum percentage grade required on a final exam.

- c = Current overall grade in class, excluding undetermined categories
- d = Desired minimum overall grade in class
- w = Weight of the final exam category

Additionally, it was assumed that the only remaining undetermined category was the final exam category. The result is produced as follows:

$$\frac{d - (100\% - w)c}{w}$$

Values are assumed to be percentages between 0 and 1, with the exception of certain variables where values greater than 1 are reasonable. It is understood that 100% is equivalent to a value of 1.0 when used for calculations. Output values are rounded to the nearest hundreth of a percent.

3 Extended Calculators

Over the summer of 2013, several additional calculators were added to the Final Grade Calculator. They were chosen from the most frequently asked questions in the comments section.

3.1 Overall grade after final exam

A new calculator was created titled "Already took the final. What is my overall grade?". Given the following variables, it would calculate the overall course grade after the final exam grade was determined.

- c = Overall grade in class before the final exam, excluding the final exam
- f = Grade achieved on final exam
- w = Weight of the final exam category

The result is produced as follows:

$$(100\% - w)c + wf$$

3.2 Grade required on the final test

Several courses appear to substitute the final exam for an additional test. A new calculator was created titled "My final counts as a test. What do I need to get?". Given the following variables, it determines the minimum percentage grade required on a final test.

- c = Current overall grade in class
- d = Desired minimum overall grade in class

- $w =$ Weight of the test category
- $n =$ Number of tests taken
- $a =$ Weighted average grade of all tests in test category
- $t =$ Number of tests the final test is worth

This calculator makes a few assumptions. First, there can be no undetermined categories in the grade, such as an undetermined participation category. Second, tests are all equally weighted, except perhaps the final test.

If tests are not equally weighted, the calculator can still be used in a roundabout way. The variables n and t are always used in conjunction, so only the ratio $\frac{t}{n+t}$ needs to be correct. Replacing n with the number of total possible test points available and t with the number of test points on the final test would also yield an accurate result. In the ideal case, this would mean scaling both the numerator and denominator of the critical ratio by a constant factor, which will not change the result.

The result is produced as follows:

$$\text{let } \tau = \begin{cases} 0, & \text{if } w = 100\% \\ \dfrac{c - wa}{100\% - w}, & \text{if } w < 100\% \end{cases}$$

The variable τ refers to the weighted average of all grades received in non-test categories. These grades are assumed not to change as a result of the final test. This definition can be generalized to any category so long as the final assignment belongs in the chosen category.

It then follows that:

$$d \geq (100\% - w)\tau + w\frac{an + tx}{n + t}$$

$$x \geq \frac{(d - (100\% - w)\tau)(n + t) - wan}{wt}$$

When $w < 100\%$, the expression on the right-hand side simplifies to:

$$\frac{(d - wa + c)(n + t) - wan}{wt}$$

When $w = 100\%$, the expression becomes:

$$\frac{d(n + t) - an}{t}$$

This is expected, because now tests are the only remaining category, and the final test can be treated as its own undetermined category with weight $\frac{t}{n+t}$.

3.3 Grade required on a multi-part final

A variation to the final test problem, some courses involve a multi-part final. A new calculator was created titled "There are 2+ parts to my final. What do I have to get on them?". Given a variable number of inputs, it determines what minimum percentage grade was required on the remaining parts of the final.

- $c =$ Current overall grade in class, not counting any part of the multi-part final
- $d =$ Desired minimum overall grade in class
- $w =$ Total weight of the final

- n = The total number of parts on the final.
- n_t = The number of parts on the final that have been taken.
- w_k = The number of points on part k of the final.
- c_k = The number of points scored on part k of the final.

The sizes of sequence w_k and c_k are n and n_t respectively. The solution to the multi-part final problem is to apply the simple final exam formula twice, using the result of the first calculation as the variable d of the second calculation. The result is produced as follows:

$$\text{let } W = \frac{\sum_{i=1}^{n_t} w_i}{\sum_{i=1}^{n} w_i} \text{ and } C = \frac{\sum_{i=1}^{n_t} c_i}{\sum_{i=1}^{n_t} w_i}$$

These two variables W and C represent the worth of the remaining parts of the final, as a percentage of the whole final, and the weighted average score on the parts of the final that have been taken. The result can then be calculated with:

$$\frac{\frac{d-(100\%-w)c}{w} - (100\%-W)C}{W}$$

This equation will fail when either $w = 0$ or $W = 0$, both of which correspond to illogical scenarios. If final exam components are weighted by percentage instead of points, input variables need to be adjust accordingly. For example, a 2-part final whose components are worth 40% and 60% can be interpreted to represent two components worth 40 and 60 points respectively. A score of 85% on the first component would translate to $40\% \times 85\% = 34$ points.

3.4 Weight of the final on a points system

By request, an additional calculator was added to calculate the weight of the final exam within a points system. Given the following variables, it determines how much the final exam is worth as a percentage of the entire grade:

- p = Total number of points possible, including the final
- w = Number of points the final is worth

The result is produced as follows:

$$\frac{w}{p}$$

3.5 Grade on the final if tests are dropped

This question is a generalization of some of the previous questions. It accounts for dropped tests, a final exam that counts in the test category, and a normal final exam category.

- c = Current overall grade in class
- d = Desired minimum overall grade in class
- w_t = Weight of the test category
- n_t = Number of tests taken

- n_d = Number of lowest test grades dropped
- a = Weighted average grade of all tests in test category
- ℓ_k = Lowest kth test grade
- n_f = Number of tests the final test is worth (can be 0)
- w_f = Weight of the final exam category (can be 0)

We first recognize that the current grade c is made up of 3 components:

- The tests that will be dropped from the grade
- The tests that will not be dropped from the grade
- The non-test part of the grade

The resulting graade will be made up of 3 different components. The first component mentioned above is removed, and a new final exam is introduced.

- The tests that will not be dropped from the grade
- The non-test part of the grade
- The final exam

To solve for the grade needed on the final exam, we need to express the non-dropped tests and the non-test grades in terms of their worth in the overall resulting grade.

3.5.1 Non-dropped tests

We can express the average grade of the non-dropped tests as:

$$\frac{n_t a - \sum_{k=1}^{n_d} \ell_k}{n_t - n_d}$$

This score will factor into the final grade as part of the test category, but it will not be the entire test category, if $n_f > 0$. So, we can say that the non-dropped tests have weight:

$$w_t \frac{n_t - n_d}{n_t - n_d + n_f}$$

To get the total worth of the non-dropped tests, we need to multiply these together.

$$\frac{n_t a - \sum_{k=1}^{n_d} \ell_k}{n_t - n_d} w_t \frac{n_t - n_d}{n_t - n_d + n_f}$$

If $n_t = n_d$, then this component of the overall grade should have 0 value. However, we can see that the above expression can be simplified to:

$$(n_t a - \sum_{k=1}^{n_d} \ell_k) w_t \frac{1}{n_t - n_d + n_f}$$

We should see that $n_t a - \sum_{k=1}^{n_d} \ell_k = 0$, if the user's numbers are correct, but the calculator chooses to ignore this term entirely if $n_t = n_d$, because there is no meaning to this term if all the tests are dropped (what kind of class does that anyway?).

3.5.2 Non-test grades

We do not explicitly ask the student for their non-test grades, but we can derive this value based on their current grade. In many cases, students have multiple categories that make up their non-test grades (classwork, quizzes, etc), so it would not be feasible to ask for their non-test grade average.

The average of the non-test grades can be expressed as:

$$\frac{c - \frac{aw_t}{1-w_f}}{1 - \frac{w_t}{1-w_f}}$$

The non-test grades have weight $(1 - w_t - w_f)$, so we can express the value of the non-test grades as:

$$\frac{c - \frac{aw_t}{1-w_f}}{1 - \frac{w_t}{1-w_f}}(1 - w_t - w_f)$$

This can be simplified to:

$$c(1 - w_f) - aw_t$$

If $w_t + w_f = 1$, then there are no non-test grades, and this category should be ignored. In such a case, we should see that $c(1 - w_f) - aw_t = 0$, but the user's inputs may have errors, so we choose to just ignore this term in such a case.

3.5.3 The final exam

The final exam gets a weight of w_f from the final exam category and a weight of $w_t n_f/(n_t - n_d + n_f)$ from the tests category. This gives us a total weight of:

$$w_f + \frac{w_t n_f}{n_t - n_d + n_f}$$

3.5.4 The formula

To derive the final formula, we first express the overall grade as a sum of the 3 components from before:

$$d = (n_t a - \sum_{k=1}^{n_d} \ell_k) w_t \frac{1}{n_t - n_d + n_f} + c(1 - w_f) - aw_t + f(w_f + \frac{w_t n_f}{n_t - n_d + n_f})$$

In this equation, we've introduced a new variable, f, to represent the final grade. To solve for f, we isolate it:

$$f = \left(d - (n_t a - \sum_{k=1}^{n_d} \ell_k) w_t \frac{1}{n_t - n_d + n_f} - (c(1 - w_f) - aw_t) \right) / (w_f + \frac{w_t n_f}{n_t - n_d + n_f})$$